FRAMING DISCOURSE FOR OPTIMAL LEARNING IN SCIENCE AND MATHEMATICS

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A physics problem asks about how much longer it will take a train to go from point A to point B when it must make a two minute stop at a station in between, than it would take with no stop. This invokes an image with a storyline that could go something like this: a diesel locomotive, pulling passenger cars, is traveling down a railroad track at some cruising speed. At some instant, it begins slowing until it comes to a stop. It remains at a standstill for two minutes while it discharges or takes on passengers. Finally, it pulls away from the station and accelerates back up to its cruising speed, continuing its journey from point A to point B. This scenario or "script" is compared to a train that just cruises on down its track at a constant speed without stopping at the station.

In a traditional problem solving approach, the student chooses and encodes the information she needs to extract from this contextual frame with identifiers (i.e., cruising speed, acceleration rates, and the length of time the train is stopped), selects formulas that will allow her to compute time in the two different cases, and maps her data onto the appropriate variables in the formulas. She performs the mathematical operations that the formulas prescribe and presto! Out pops two answers. If she is adept, she will be able to compare the two answers she has computed and connect them back to the problem statement.

Students who adopt the procedural approach to this problem described above find it very difficult, but when the conceptual structure is mapped onto a spatial representation, even a mental one, it becomes a relatively straightforward task. If, in addition, the student can produce an inscription that encodes spatial information comparing the two journeys, they may be able to manipulate it to arrive at a solution without resorting to the multipart algebraic solution described above. In the next excerpt, Zane uses a graph, reinterpreting it at several points, to help Gui, who prefers to rely on a procedural approach to problem-solving, navigate through the problem space for the train problem described above.

> *Zane: First, you've got to find out how long the whole station thing takes. Did you figure that out?*
>
> *Gui: What formula do you use?*
>
> *Zane: There's no formula that you use. You have to think about this one. Let's think about it…it says… (looks back at the problem statement in the book)…the train decelerates at a uniform rate of one meter per second.*
>
> *Gui: You need to know how long it takes.*
>
> *Zane: It doesn't matter.*
>
> *Gui: But…*
>
> *Zane: It does not matter. Look. Get this number down to meters per second first. (Points to 72 km/h).*
>
> *Gui: But…*

Zane: Get it down to meters per second. (After some hesitation, Gui writes 72,000 m/h on her paper, then picks up a calculator, and starts to press buttons.)

Gui: Is it this? (She holds up her calculator for him to see the answer.)

Zane: No. Okay. Alright. (Takes out a sheet of paper) You've got 72 kilometers per hour to meters per second. Alright? Okay. No, we'll get it to meters first. So…one thousand meters over one K-M equals 72,000 meters for an hour, okay? And then so I want to find…and so there's… in one hour there's sixty minutes, so I divide by 72000 by sixty alright and then there's 60 seconds in one minute. Twenty meters per second. Do you get how I got that? (Gui nods.) So if it's decelerating at one meter per second squared how many seconds is it going to take it to decelerate?

Gui: Twenty.

Zane: Twenty seconds. That's correct. So it helps to draw a graph. Right here it's cruising at 20 meters per second. Then it slows down (talks as he sketches a velocity time graph for Gui) and then here's that 20 seconds of decelerating right here (draws a diagonal line down to the t-axis) and then it stops for 2 minutes. So how many seconds is two minutes?

Gui: Two minutes?

Zane: It says it stopped for two minutes.

Gui: One hundred twenty?

Zane: One hundred twenty. So what's one hundred twenty plus twenty?

Gui: One forty?

Zane: One hundred forty. There. (He continues to draw the graph.) And then…. (looks back at the book for a moment) and then it accelerates at point five meters per second squared.

Gui: It comes to a stop?

Zane: Yes it does. It's stopping at a train station for two minutes. This is a time graph. Time-velocity. Okay. So it stops here for two minutes and then it can only accelerate at point five meters per second squared. So it's eventually going to make it's way back up to twenty at point five.

Gui: Point five meters per second squared?

Zane: So how long does it take to get to forty? I mean to get to twenty...if it's accelerating at point five… (he points to the graph he has drawn)

Gui: forty-five?

Zane: No, Gui. You need to get up to twenty meters a second and you're going up at one half,

> *right? (Gui picks up her calculator and waits. He thinks for a minute and finally writes 20 divided by one half. Gui keys this into her calculator.)*
>
> *Gui: Forty?*
>
> *Zane: Forty. Okay. So it takes forty seconds to do that. So what's 140 second plus 40 seconds?*
>
> *Gui: One eighty?*
>
> *Zane: That's just how long it took him to…*
>
> *Gui: One eighty?*
>
> *Zane: Yes. That's how long the stop was. Okay? And how much distance did he cover in this time? It's just the area here plus the area here. He points to the two triangular areas under the graph of the velocity time graph he has drawn for her.*
>
> *Gui: But it doesn't ask for the distance. It doesn't matter.*
>
> *Zane: It doesn't matter, but that's how you need to figure it out. No we're not done yet. (He reads from the book)…how much time is lost in stopping at the station…so we need to figure out the distance that this took, okay? So what's your…*
>
> *Gui: This plus this?*
>
> *Zane: Mm hmm. One-half base plus height. (He goes and borrows a calculator from someone at another table and then brings it back and computes the areas of the two triangles; 200 meters and 400 meters respectively.) So how much is that…200 plus 400?*
>
> *Gui: six hundred?*
>
> *Zane: Right. So that's how far the train went while it was stopping. The other train was going 20 meters per second. So how many seconds it going to take for it to go 600 meters?*
>
> *Gui: (divides on her calculator) Thirty seconds?*
>
> *Zane: Thirty seconds. So it took [the other train] thirty seconds to cross this whereas it took this one 180 seconds to stop. So how much time is lost?*
>
> *Gui: Is it this minus this?*
>
> *Zane: …and that gives you one hundred fifty seconds…*
>
> *(Gui nods slowly)*
> *(DHS 10-3-05)*

In this episode, Zane is clearly reprising for Gui the way he reasoned through this problem with the aid of a graph. When she asks him for a formula at the outset, he tells her that there is no formula—she just needs to figure it out by thinking about it. His tool for thinking in this instance was a spatial representation. Although it is not clear at the end of this episode that Gui has learned to do the inscription-assisted reasoning in which Zane is engaging, she appears to be able to follow his lead as he takes her down the path he followed through this problem space. It is clear that Zane is able to read many more things from his graph than just velocity and time. He maps the train's journey first as a function of time, interpreting acceleration as a change in the train's velocity, recognizing that the area under his graph represents displacement, and comparing the time intervals it took each train to experience the same displacement. Initially he uses his graph to communicate about the train's journey through time. Once he has accomplished this he uses the same graph to show its journey through space.

An inscription has the additional benefit of being shareable and transportable. Certain inscriptional practices, such as graphing, motion mapping and force diagramming, are well enough developed that the inscriptions they generate are adaptable and reusable to represent a variety of spatial relationships.

Metaphors used in the mapping of conceptual structure to a spatial representation can shift as well. Here Hannah talks about a graph, while she constructs it, as if it is a map from which she is reading information:

> *Hannah: Okay so, just to give another example, if you had another graph like this and you're A line, like, went up like this, and then your B line still did… (camera zooms in on drawing) I guess it doesn't really matter. Like if your B line was like that then you'd still be looking at your intersection point meaning you'd be taking these distances. So it doesn't matter.*
>
> *(DHS 10-5-05)*

Seconds later Zane is looking at the same graph and talking about it as if it were a journey:

> *Zane: if we put this on the v vs. t route you'll notice, (pause, drawing) this isn't so accurate, it should be going like this, and then this line, and then so…*

Creating and interpreting these graphical and diagrammatic images is a skill that is learned, and in the learning process, students must develop an awareness of how the choices they make in constructing a representation can be interpreted. There is a risk that the task of producing pre-determined inscriptions can become so proceduralized that the inscription becomes an end in itself and it is not employed as a reasoning tool.

> *Hannah: Okay, so for the first page we did everything pretty much together except for the motion map. What did you guys do for the motion map?*
>
> *Gui: I got this. Oh, I forgot my dots.*
>
> *Heather: Like how many, see we all have different things. How many tick marks did you put?*
>
> *Jimmy: I messed up. (Pause) Oh the first page?*
>
> *Zane: How many dots?*
>
> *Hannah: I have, well I just put 10.*
>
> *Jimmy: Well I didn't use tick marks because you don't really know the velocity.*
>
> *Hannah: but does it matter if you use them?*
>
> *Jimmy: (makes a questioning gesture with hand)*
>
> *Hannah: So did you show that the lines got smaller?*

Jimmy: Because you don't really have quantitative evidence…

Hannah: So you just make sure that the lines got smaller?

Jimmy: yeah because it was a negative acceleration. After (counting) seven…

Gui: You didn't have to do that.

Hannah: I just assumed that you could put a bunch of them. Well does it matter how many you have I mean do I have to do it by…

Gui: It doesn't matter.

Hannah: I think you're supposed to put ten.

(DHS 10-5-05)

In the previous episode, Hannah's goal was clearly to make the motion map correctly. At no point in the conversation did anyone express any intention of using their inscription to help them figure something out about the motion of the object they were trying to represent.